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Method for calculating the stress intensity factor for mode-i indentation with eccentric loads

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Abstract. Stress intensity factor is one of the most important parameters in fracture mechanics. The singularity and distribution of Mode-I indentation stress field are the same with the cracks in solids. Based on the conservation law, a new method to calculate stress intensity factor for Mode-I indentation with eccentric loads is proposed. Compared with the method reported in the literatures, the proposed method can give two different stress intensity factors at both ends of the flat-ended square rigid indenter and is simple in calculation.

Key words. Stress intensity factors, plane indentation, conservation law, crack.

1. Introduction

The flat tooth cutting tool of brittle materials, such as rock, ceramics and concrete, is a typical application of plane indentation. The indentation model was first used in the macro/micro fretting fatigue, abrasion study, elastic modulus test technique, etc. Among others[1], gave the impacts of the parameters controlling the finite element simulation of instrumented indentation tests and their consequences on numerical results are reviewed in case of homogeneous materials and functionally graded ones,[2] used systematic finite element analysis to examine that how indentation creep tests can be employed to retrieve the steady-state creep parameters pertaining to regular uniaxial loading,[3] used the finite element analysis to study the influence of crack forms on indentation hardness test of ceramic materials.

An important part of fracture mechanics is to calculate the stress intensity factor.

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No matter for fracture theory or engineering application, it is very important to find a simple method to solve the stress intensity factor. Similar to the crack problem, the stress intensity factor is the only measure of the singular stress field, and also a key parameter to establish the fracture criterion of indentation fracture. The relationship between the energy release rate of the indentation cracking, cracking direction and stress intensity factors for Mode-I indentation[4], and stress intensity factors of Mode-I indentation with positive loads are got. In this paper, based on the conservation law, further research will be made about the stress intensity factors for Mode-I indentation with eccentric loads. Considering that the two ends of the rigid square-ended punch have different stress conditions, therefore, two kinds of stress intensity factors are given. This makes the study of Mode-I indentation problem more perfect, and it is also very useful for the study of rock breaking technology.

2. Plane indentation structure and singular stress field

The indentation geometry is illustrated in Fig. chmetcnvUnitNameaSource-Value1HasSpaceFalseNegativeFalseNumberType1TCSC01a. A rigid square-ended punch of width chmet cnv Unit Name l Source Value 2 Has Space False Negative False Number Type 1 TCSC02l is pressed by a normal load, P, onto the surface of a frictionless haif-plane, having a Poisson′s ratio u and elastic modulus E. According to the classical analysis, the stress distribution of the indentation interface is shown as follows[5]:

$$p(x_1) = -\frac{P}{\pi\sqrt{l^2 - x^2}} \tag{1}$$

When a change is made about the co-ordinates $x = r - l$, and the solution is expanded, for small$r$, using binomial distribution, the pressure in the neighborhood of the punch corner varies as:

$$p(x_1) = -\frac{P}{\pi}\left(\frac{1}{\sqrt{2lr}} + \cdots\right) \tag{2}$$

The contact problem of a rigid, square-ended indenter and elastic solids is the two-dimensional plane indentation problem. Whilst the requirement that the contact be frictionless means that the shear traction is zero everywhere along the boundary. This means the boundary conditions along the line x_2=0 are identical in the contact and crack problem as shown in Fig. 1b. Using the polar coordinates (r,)the stress at the plane indentation is given by a classical crack-tip field solution[6],

$$\begin{pmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{r\theta} \end{pmatrix} = -\frac{K_{\mathrm{I}-ind}}{\sqrt{2\pi r}} \begin{pmatrix} \cos\frac{\theta}{2}\left(1 + \sin^2\frac{\theta}{2}\right) \\ \cos^3\frac{\theta}{2} \\ \sin\frac{\theta}{2}\cos^2\frac{\theta}{2} \end{pmatrix} \tag{3}$$

The stress field is singular stress field, and the definition of K_{I-ind} is "indentation stress intensity factor", which is expressed as follows:

$$K_{\mathrm{I}-ind} = \frac{P}{\sqrt{\pi l}} \tag{4}$$

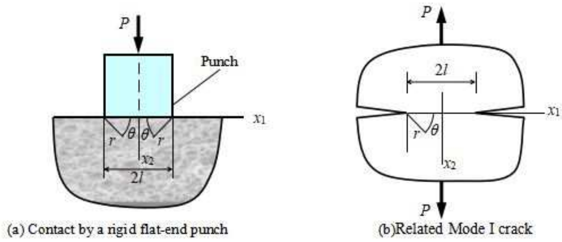

Fig. 1. Contact between atwo-dimensional retangular punch and a substrate

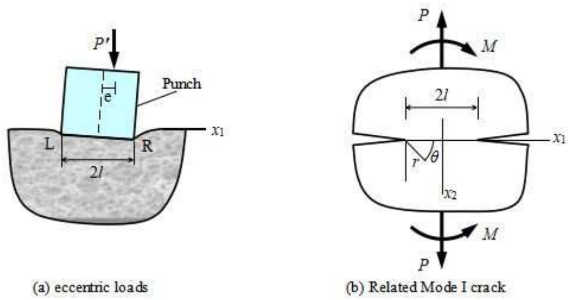

Fig. 2. Contact between a two-dimensional rectangular punch of eccentric loads and a substrate

For Eq.(3), this result will be the same as that for a plane semi-infinite crack (with an uncracked ligament of $2l$) loaded remotely with a normal load P, if the negative sign is changed into positive, as shown in Fig. 1. Then the plane indentation can be treated in comparison with a crack growth problem with a well-defined stress singularity, which means that the plane indentation (FigchmetcnvUnitNameaSourceValue.1HasSpaceFalseNegativeFalseNumberType1TCSC0.1a) should be treated as the akin mechanism of fracture similar to the mode-I crack (Fig.1).

When a flat surface of the elastic solids is eccentrically loaded by a relatively rigid, square-ended punch, as shown in Fig. 2, eccentric loads are equivalent to positive pressure and bending moment. In passing, it may be emphasized that the left corner of indenter never leave the elastic solids, then the left corner of indenter and the right corner of indenter arise two different Mode-I singular stress fields, both of them are similar to Eq.(3).

3. Conservation integral

Based on the two-dimensional conservation law, for a closed integration path without any crack and cavity in it, the following integrals are given as follows[7−10]:

$$J_j = \int_s (wn_j - T_i u_{i,j})ds, j = 1, 2 \tag{5}$$

Eq.(5) has two components (J1and J2), and they can be used to calculate the stress intensity factors for the cracks in elastomers and plane indentation. In the following section, some key steps are given to calculate the stress intensity factors for the indentation based on the J2-integral.

4. Stress intensity factor for mode-I indentation with eccentric loads of finite-boundary

The problem of the stress intensity factor for the ideal smooth plane has been discussed in the literature[4]. Mode-I indentation with eccentric loads, as shown in Fig. 1, where $P^{'}$ is the eccentric loads, M is the bending moment and P is the normal load. And then the stress intensity factor for Mode-I indentation with eccentric loads can be expressed as the superposition of stress intensity factors for the bending moment and the normal load. In this paper, we take the Finite-Boundary elastic body as an example to study J_2integral, i.e.$0<x_2<h$, $0<x_1<W$ and hW as shown in Fig.3.

Under the action of concentrated force, as shown in Fig.3 (a), a closed integration path $S_{abcdefghija}$ is selected, and then from Eq.(5), following results can be given[7−9]

$$J_{2--P} = \oint_{S_{abcdefghija}} (wn_2 - T_i u_{i,2})ds = 0 \tag{6}$$

For the path S_{bc} and S_{de}, because T_i=0 and n_2=0, and the following result was found:

$$J_{2-P} = \int_{S_{bc}} wn_2 - T_i u_{i,2})ds = \int_{S_{de}} (wn_2 - T_i u_{i,2})ds = 0 \tag{7}$$

When S_{gh} and S_{ij} are straight lines, S_{fg} and S_{ja} account for a quarter of a circle, and S_{ija} and S_{fgh} are within the K-dominant regions, it is not difficult to get the following results:

$$J_{2-P} = \int_{S_{gh}} wn_2 - T_i u_{i,2})ds = \int_{S_{ij}} (wn_2 - T_i u_{i,2})ds = 0 \tag{8}$$

$$J_{2-P} = \int_{S_{fg}} wn_2 - T_i u_{i,2})ds = \int_{S_{ja}} (wn_2 - T_i u_{i,2})ds(planestrain) \tag{9}$$

Where E is the elastic modulus and μ is the Poisson's ratio, and the cross sectional area is relatively far from the contact zone, the following expression can be given by

$$J_{2-P} = \int_{S_{cd}} wn_2 - T_i u_{i,2})ds = \bar{w}^- + P\tilde{u}^-_{2,2} \quad (10)$$

$$J_{2-P} = \int_{S_{hi}} wn_2 - T_i u_{i,2})ds = -(\bar{w}^+ + P\tilde{u}^+_{2,2}) \quad (11)$$

Where the $_i$ manifests the displacement of the neutral axis, the superscript "-" denotes the remote uncracked cross section. The i can be obtained by elementary strength theory of materials. w is the strain energy density per unit length. Then substituting Eqs. (7)—(11) to Eq.(6), we have

$$J_{2-P} = \frac{1-\mu^2 K^2_{\mathrm{I}-ind-P}}{\pi E} + 2\int_{S_{ab}} w_P ds = (\bar{w}^- + P\tilde{u}^-_{2,2}) - (\bar{w}^+ + P\tilde{u}^+_{2,2}) = \frac{P}{2}(\tilde{u}^-_{2,2} - \tilde{u}^+_{2,2}) \quad (12)$$

The axial strain $\tilde{u}^-_{2,2}$ and $\tilde{u}^+_{2,2}$ have been found:

$$\tilde{u}^-_{2,2} = -\frac{P}{EA} \quad (13)$$

$$\tilde{u}^+_{2,2} = -\frac{P}{EA}\int_0^1 \frac{d\xi}{1-(2a/W)\sqrt{1-\xi^2}} \quad (14)$$

In equation above, $A = W$ indicates the remote cross section area, because of the movement and cracking of indentation boundary, and $S_{ab}$and S_{ef} are out of the K-dominant regions. So the integral in left-hand side of Eq.(12) is a small quantity, which can be neglected. Substitution of Eqs. (13) and (14) into (12) will yield the following:

$$J_{2-P} = \frac{(1-\mu^2)K^2_{\mathrm{I}-ind-P}}{\pi E} = \frac{P}{2}(\tilde{u}^-_{2,2} - \tilde{u}^+_{2,2}) = (\frac{P}{EW}\int_0^1 \frac{d\xi}{1-(2a/W)\sqrt{1-\xi^2}}) \quad (15)$$

The stress intensity factor in the situation of stress concentration is obtained as follows:

$$K_{\mathrm{I}-ind-P} = \frac{\sqrt{\pi}P}{\sqrt{2W1-\mu^2)}}(\int_0^1 \frac{d\xi}{1-(2a/W)\sqrt{1-\xi^2}} - 1)^{1/2} \quad (16)$$

Another situation is the bending moment concentration, a closed integration path $S_{abcdefghija}$ is still selected, as shown in Fig.3(a), and according to the conservation law and referring to the above calculation methods, we have:

$$J_{2-M} = \frac{1-\mu^2 K^2_{\mathrm{I}-ind-M}}{\pi E} + 2\int_{S_{ab}} w_M ds = (\bar{w}^- - M^-\varphi'^-) - (\bar{w}^+ - M^+\varphi'^+) \quad (17)$$

$M\!=\!eP$, as shown in Fig. 1(c), and this will yield the following:

$$K_{I-ind-M}=\frac{\sqrt{\pi}P}{\sqrt{2W(1-\mu^2)}}\Big(\int_0^1\frac{12\times\frac{\varepsilon}{W}^2}{1-{}^{2a}\!/_{W}\sqrt{1-\xi^2})^3}d\xi-12\times(\frac{e}{W})^2\Big)^{1/2} \tag{18}$$

There are two unsymmetric Mode-I singular stress fields, because of eccentric loads. In this paper the two different singular stress fields are defined as L-Mode-I singular stress fields and R-Mode-I singular stress fields. Then, according to Eqs.(16) and (18), we find

$$K_{\mathrm{I}-ind-R}=\frac{P\sqrt{\pi}}{\sqrt{2W1-\mu^2}}$$
$$\left[\int_0^1\frac{1}{(1-{}^{2a}\!/_{W}\sqrt{1-\xi^2}}+\frac{12\times(\frac{e}{W})^2}{(1-{}^{2a}\!/_{W}\sqrt{1-\xi^2})^3}d\xi-1-12\times(\frac{e}{W})^2\right]^{1/2} \tag{19}$$

$$K_{\mathrm{I}-ind-L}=\frac{P\sqrt{\pi}}{\sqrt{2W1-\mu^2}}$$
$$\left[\int_0^1\frac{1}{(1-{}^{2a}\!/_{W}\sqrt{1-\xi^2}}-\frac{12\times(\frac{e}{W})^2}{(1-{}^{2a}\!/_{W}\sqrt{1-\xi^2})^3}d\xi-1+12\times(\frac{e}{W})^2\right]^{1/2} \tag{20}$$

Here a hypothesis is given by $e/l\!=\!0.3$, so the following is obtained:

$$\frac{e}{W}=0.3(0.5-\frac{a}{W}) \tag{21}$$

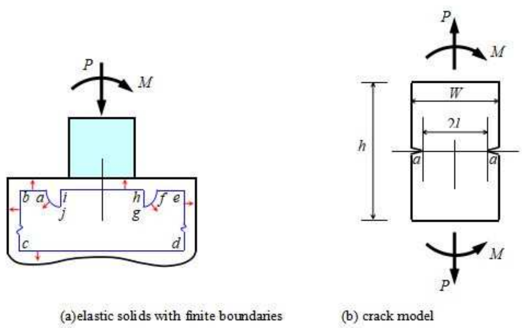

Fig. 3. Contact model for elastic solids with finite boundary and crack model

5. Conclusions

In this paper, we propose an approach of calculating Mode-I plane indentation with eccentric loads based on the conservation law. The results are in agreement with the superposition of stress intensity factors for the bending moment and the normal load. The results also indicate that the boundary cracking of Mode-I plane indentation with positive loads and eccentric loads have different critical cracking, and they are different from the crack growth problem, but they all have the same asymptotic singular stress field, the same mechanical essence, and homologous cracking mechanism. In future work, its theoretical significances covers propose a new fundamental theory on the fracture-based rock breakage. Establish the theoretical basis for rock-like and brittle materials cutting and the design method of oil and gas cone bit.

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